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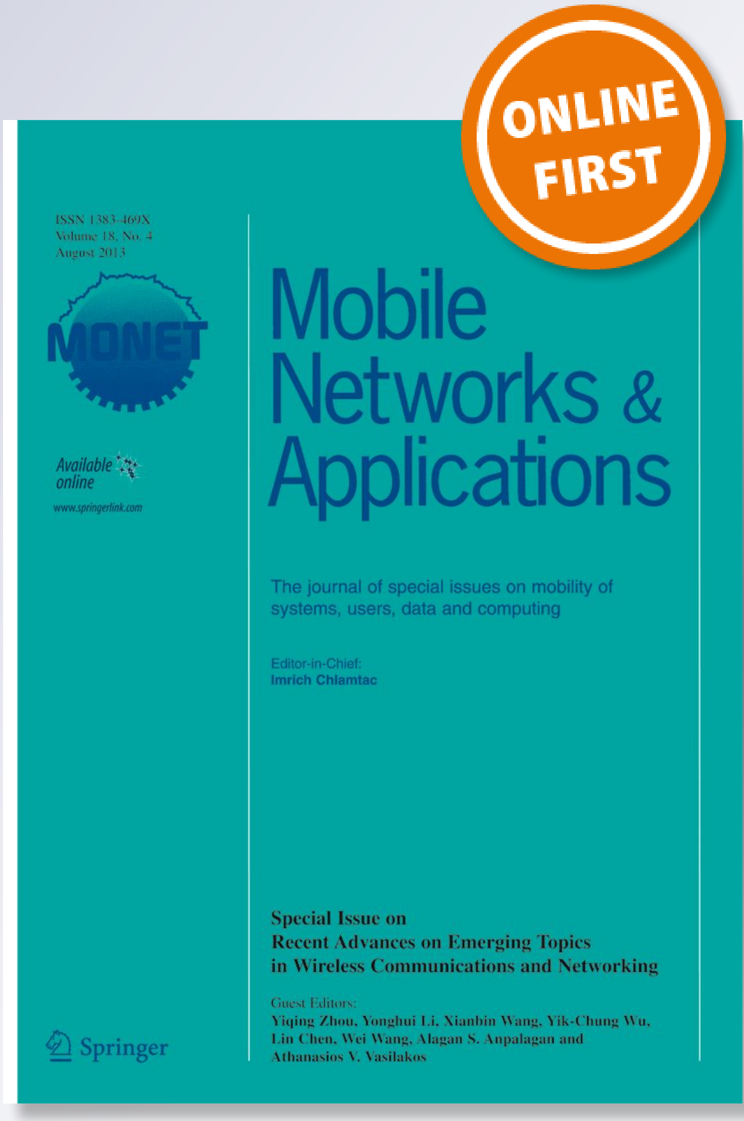

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Outage Probability Analysis for Energy Harvesting Cooperative Relays in a Clustered Environment

Mateen Ashraf[1] · Ju Wook Jang[2] · Kyung-Geun Lee[1]

Abstract This paper presents the outage probability analysis of the energy harvesting (EH) decode and forward (DF) cooperative relay network when more than one relays are available to assist the communication between source and destination in the presence of the direct connection. The relays use power splitting (PS) protocol with adaptive PS ratio for EH. As wireless EH can be more beneficial over smaller distances therefore a clustered environment is considered in which the source, destination and relays are located in a small area. First, we analyze the performance of selection cooperation (SC) which requires channel state information (CSI) at the source. High signal to noise ratio approximation of the outage probability is provided for this case. Secondly, we present the performance of all relays cooperation (ARC) scheme which requires no CSI at the source. Lower and upper bounds of the outage probability are presented for smaller number of relays in ARC scheme whereas high signal to noise ratio approximation is provided for higher number of relays. Simulation results validate the analytical results and show that SC scheme outperforms ARC scheme at the expense of CSI requirement.

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Keywords Decode and forward · Energy harvesting · Outage probability · Power splitting

1 Introduction

Cooperative communications has been shown to be a very attractive method for improving the performance of wireless networks. In cooperative communication a number of nodes which are present in the network can assist the communication between source and destination. These assisting nodes are generally referred to as relays. The relays can use amplify and forward (AF) protocol or decode and forward (DF) protocol for forwarding. In AF the relay only amplifies the received signal and relays the amplified version of the received signal to the destination. On the other hand in DF protocol the relay first decode the information and then transmit a new copy of the signal to the destination. In conventional cooperative communication the relays use their own energy for relaying purposes. This can exhaust their battery very quickly. One other attractive technique is to harvest energy wirelessly and use it to forward the information to the destination. The use of EH can enhance the battery life of the cooperating relays. In this way the life time of the network can be improved. In this context, energy can be harvested from the environment, for example solar, vibration, thermoelectric effects and other physical phenomenon [1]. However, these sources of energy may not provide constant power. One other approach is to harvest from RF transmissions. This technique is known as wireless powered communication (WPC). Three types of WPC architectures are considered in the literature [2]. These are wireless powered transfer (WPT), wireless powered communication network (WPCN) and simultaneous wireless information and power transfer (SWIPT). In the first two

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architectures a dedicated RF transmitter wirelessly provides the power to EH capable devices. In the third architecture the source of information provides the power to the EH device. Two types of EH protocols for cooperative relays are discussed in the literature when energy is harvested from the source of information. The first is the time switching (TS) and the second is power splitting (PS). In TS the relay relay divides the receiving time into two portions. In the first portion the relay harvest energy and in the latter portion the relay performs the information processing. In the PS scheme, during the receiving time, the relay directs some portion, α, called the power splitting ratio, of the signal to the EH circuitry, while the remaining portion is directed towards the information processing circuitry. The value of α can affect the performance of the system because if too small α is chosen then very small energy is available for retransmission while on the other hand if $\alpha = 1$ then very small power is available for the information processing at the relay and decoding maybe unsuccessful at the relay.

In [3] the authors have considered amplify and forward (AF) relaying with both the TS and PS EH protocols. They have considered fix values of the TS ratio and PS ratio for EH in their mathematical analysis. Analytical expressions for outage probability and ergodic capacity are provided in their work. In [4] the authors have discussed DF relaying with the PS protocol. The optimal value of the PS ratio is used to maximize the performance in terms of outage probability. In their work, they have considered the possibility of more than one source destination pair. However, both of these works assume no direct connection between the source and the destination. This assumption is less likely to be true if the source and destination are located nearby each other. A dynamic role selection mechanism for three node network is proposed in [5] where AF protocol is used by the relay. It is shown that considerable improvement in terms of outage probability is possible if the roles of source, relay and destination can be dynamically adjusted according to the channel conditions. Stackelberg game approach is proposed in [6] to incentivize the relays for signal and energy cooperation. Time splitting is used to harvest energy from the source while source can adjust its transmit power according to the policy adopted by the relay. The relay uses the harvested energy to retransmit the source information as well as its own information. The objective of the game is to maximize the total throughput of the network. Rate and energy tradeoff for cognitive cooperative network with EH is analyzed in [7]. Analytical expressions for outage probability are provided for the primary and secondary network. Optimization problems are formulated to maximize the energy and spectrum efficiency of primary and secondary networks. In [8] the author provide the achievable throughput for the cognitive radio network. It is shown that the throughput is unaffected by the channel access probability and depends on the channel sensing probability. Average packet loss probability and average delay for the EH overlaid wireless sensor networks are provided in [9]. Similar analysis is carried out in [10] however the authors have also considered the sensing energy in addition to the transmission energy of the sensor while analyzing the delay performance of the sensor network. Direct and cooperative communication in randomly deployed dense sensor networks are analyzed in [11]. It is shown that cooperative communication can improve the life time of the network however the communication performance is better for the direct communication.

1.1 Related work

When multiple EH relays are available the optimal strategy in terms of resource allocation is to select the best relay among the available relays. In this regard, [2, 12–21] have studied the outage probability in the case of multiple relays. In [2] the author shows the outage probability of the EH cooperative network when relay can either harvest energy or decode and forward the received signal. The performance of multiple AF EH relays is analyzed in [12]. However, the energy is harvested from ambient renewable resources and not from the source of information. In [13] the authors have found the outage probability for DF cognitive cooperative network with EH, however they have considered fix value for the PS ratio in their work. Similar work has also been carried out in [14] however for nonidentical Rayleigh fading channels. The direct connection is not considered in their work also and the PS ratio is assumed to be constant for the entire communication period. As CSI is generally assumed to be available at the destination [1] hence it is beneficial to change the PS ratio in accordance with the channel condition between the source and relay. The value of PS ratio can affect the performance of the system [3]. The Battery aware relay selection cooperation scheme is discussed in [15–17], however they have assumed no direct connection between the source and destination. A tradeoff perspective between energy transfer and information transfer is provided in [18] when multiple number of relays are available to assist source to destination communication in absence of direct connection. The relays do not harvest energy from the source instead they provide wireless energy to a group of EH nodes. In [19] the authors assumed direct connection between the source and destination however the distance between the source and destination is assumed to be much larger than the distance between relays and destination. Further, the relay selection is based on the distance between relay and source instead of the instantaneous CSI. Relay selection with causal and non causal CSI availability are considered in [20]. AF relaying protocol is considered in their work and no direct connection is available between source and destination. Further, the selected relay is assumed to be transmitting at constant power which implies that relays can use their own energy for transmission or that the energy harvested in past time slots can be used in future time slots. Further, in the clustered environment it is less likely that the source and destination do not have a direct connection. Therefore, it is necessary to consider the direct connection also in clustered environment. An opportunistic relay selection scheme is considered in [21] where multiple source to destination pairs competes for the EH relays selection. In their work the relay harvest energy from the surrounding environment.

1.2 Motivation and contribution

In this paper we analyze the outage probability of the EH relay cooperative network when the source, destination and relays are clustered in a small area. Secondly, we assume that the relays do not use any of their own energy for retransmitting the source signal to the destination. The first assumption of clustered environment can be justified with the fact that wireless power transmission loss increases exponentially with distance and mobile nodes transmit at smaller transmit power than the base station transmit power. Hence, EH cooperative relaying is more beneficial when performed over smaller distances rather than at longer distances. In addition, in the clustered environment it is less likely that cluster nodes do not have direct connection among each other. It is very much possible that relays do not act altruistically and need the source to power their retransmissions for the source. Hence, the second assumption is also justified.

The main contribution of the paper is the analysis of the outage probability for the EH cooperative network where relays use only harvested energy from source for forwarding the signal to the destination. In particular, we have presented high SNR approximations and bounds for two different cooperation schemes with any number of relays. Our work is different from the above discussed literature in terms of (i) availability of direct connection, (ii) DF relaying, (iii) instantaneous CSI plus harvested energy based relay selection and (iv) adaptive power splitting ratio for energy harvesting. To the best of our knowledge no existing work considers all these possibilities in a single system model. In the following we emphasize the importance of each of the above difference in an energy harvesting setting.

1.2.1 Direct connection availability

We have considered energy harvesting in a clustered environment where all the nodes are placed nearby each other. The use of clustered environment is considered because wireless energy losses increase exponentially with increasing distances and hence wireless energy harvesting may not be useful if preformed at longer distances. Further, as the relay use only harvested energy from the source transmission in the first time slot (for relaying source information) therefore it can be assumed that they can relay the source signal to destination over small distances only. Due to these reasons we have assumed that all the nodes are placed nearby each other. As nodes are placed nearby each other in the clustered environment therefore in the clustered environment it is very important to consider the direct connection.

1.2.2 DF relaying

We consider DF relaying because it is possible that relays forward the source information only when they are able to decode it successfully in the first time slot. Otherwise they just perform energy harvesting for their own use and do not take part in the relaying operation.

1.2.3 CSI plus harvested energy based relay selection

As different relays may harvest different amount of energy during the first time slot therefore it is very important to consider the amount of harvested energy in addition to the decoding result of the source information while selecting the best relay. This step is different from the existing works because in the existing works the relay that decodes the source information successfully and has the best channel conditions between itself and destination is selected for relaying.

1.2.4 Adaptive power splitting ratio

The power splitting ratio used in our work is not fixed. It is adapted to make sure that sufficient power is allocated to the information decoder for decoding. If we pick any other fix power splitting ratio then it is possible that more power is allocated to energy harvester and less power is allocated to information decoder (or vice versa). Hence, fix power splitting ratio may result in loss of performance. To overcome this problem we have used adaptive power splitting ratio that allocate the power to information decoder and energy harvester circuitry according to the channel condition (which is generally available at the receiver) between source and relay. In this way a relay will harvest energy for relaying operation only if it is able to decode the source information correctly. The related discussion can be found in Section 3.1.

Two schemes are analyzed: (i) Selection cooperation (SC) and (ii) All relays cooperation (ARC). Our main results are that (i) the SC scheme outperforms the ARC scheme in terms of outage probability and (ii) in ARC scheme, for small transmit powers of the source, the system with smaller relays can perform better than the system with more number of relays.

1.3 Organization

The remainder of the paper is organized as follows. Section 2 discusses system model and present the basic assumptions considered in the rest of the paper. Outage probability analysis of two cooperation schemes is presented in Section 3. Simulation results are discussed in Section 4. Finally, conclusions are drawn in Section 5.

2 System model

We consider a wireless communication system with source (S), destination (D) and M relays as shown in Fig. 1. The relays work in half-duplex manner, meaning that they can only transmit or receive at a time. The relays use harvested energy for relaying source information to the destination and they do not use any of their own energy for transmission purpose. The channel gain between node x and y is denoted by $|h_{xy}|^2$. In the rest of the paper, all the nodes are assumed to be clustered within a small area, and hence all the channel gains follow exponential distribution with parameter λ that is $|h_{xy}|^2 \sim \lambda e^{-\lambda|h_{xy}|^2}$. Here, we assume that path loss among the cluster nodes is lumped into λ and that it is same for all the channels within the cluster [22, 23]. The channels are assumed to be Rayleigh block fading which means they remain constant over one transmission time however they can have different values in different transmission times. The source transmit power is P. The total transmission time 'T' is divided into smaller equal length time slots. The number of time slots depend upon the cooperation scheme used by the source. We will consider following cooperation schemes.

- Selection cooperation (SC)
- All relays cooperation (ARC)

In the first cooperation scheme 'T' is divided into two equal length time slots, while in the second cooperation scheme 'T' is divided into $(M + 1)$ time slots. In SC, the source transmits its information to the destination in the first time slot, and in the second time slot, the source selects the best relay for forwarding the information to the destination. The selection of the best relay takes into account the CSI as well as the harvested energy at the relays during the first time slot of transmission time. In the second scheme, the first time slot will be used by the source to transmit its information to the destination, while in the remaining M slots, each relay will retransmit the decoded information during its own turn. In the first scheme, the CSI between S to all the relays and from all the relays to D is required at the source, whereas in the second scheme this is unnecessary.

3 Outage probability analysis

Before we analyze the outage probability of these schemes we show the outage probability of the case when there is no direct connection between the source and the destination and there is only one EH harvesting relay available. This result will be used to assess the performance of the other discussed schemes.

3.1 Single relay case

During the first time slot the source sends its information to the EH relay. The relay will decode the information in the

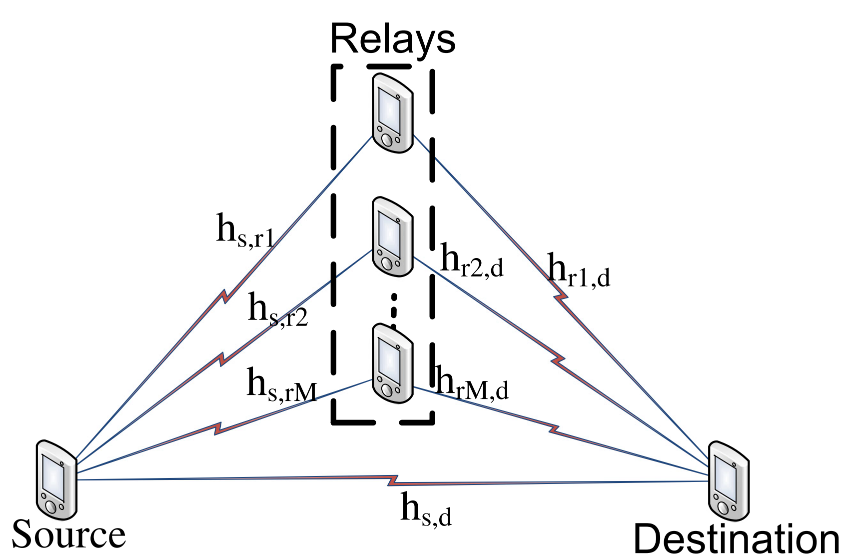

Fig. 1 System model with EH relays, source and destination

first and will also harvest energy from the received signal. As discussed above the relay uses PS protocol for EH. In this protocol during the first time slot the relay distribute the received signal into two portions. One for decoding and the other for EH. The received signal by the relay during first time slot of the n-th transmission time is given as

$$y_n = \sqrt{P} h_{s_n r} b_n + z_n, \tag{1}$$

where $h_{s_n r}$ is the channel between source and relay, b_n is the transmitted symbol and $z_n$is the additive white Gaussian noise with N_0 variance during the first time slot of the n-th transmission time. If the PS ratio is $\alpha_n \in [0, 1]$ during n-th transmission time then $(1 - \alpha_n)$ portion of the signal is sent to the decoding circuitry and α_n portion is sent to the EH circuitry. The decoding decision in the first time slot is based on following observation

$$y_{n,d} = \sqrt{(1-\alpha_n)P} h_{s_n r} b_n + z_n, \tag{2}$$

where $y_{n,d}$ is the signal sent to the decoding circuitry during the first time slot of the n-th transmission time. The signal sent to the EH circuitry will be

$$y_{n,e} = \sqrt{\alpha_n P} h_{s_n r}. \tag{3}$$

Note that we have not multiplied z_n by $\sqrt{1-\alpha_n}$ in Eq. 2 and similarly we have not included $\sqrt{\alpha_n} z_n$ in Eq. 3 [3]. By doing so we have simplified the analysis and the analysis result will be a lower bound on the performance [19]. With the help of Eq. 2 we can find the mutual information between source and relay as follows

$$I_r = \frac{1}{2} \log \left(1 + (1-\alpha_n) SNR |h_{s_n r}|^2\right), \tag{4}$$

where $SNR = \frac{P}{N_0}$. The value of α_n should be chosen such that adequate signal is forwarded to the decoding circuitry so that decoding is successful at the relay. Therefore, the optimal value of α_n for a certain $I_r = I$ can be found from above relation as

$$\alpha_n^* = 1 - \frac{2^{2I} - 1}{SNR |h_{s_n r}|^2}. \tag{5}$$

However if $|h_{s_n r}|^2 < \frac{2^{2I}-1}{P}$ then we may have a negative value for α_n. To avoid this situation we use the following value of $\alpha_n$as the optimal value

$$\alpha_n^* = max\left(0, 1 - \frac{2^{2I} - 1}{SNR |h_{s_n r}|^2}\right). \tag{6}$$

With this PS ratio the power available for retransmission during the second time slot can be found using (3) as

$$P_h = \eta \left(P |h_{s_n r}|^2 - (2^{2I} - 1) N_0\right), \tag{7}$$

where η is EH efficiency factor. The harvested power will be zero if $\alpha_n = 0$ that is $|h_{s_n r}|^2 < \frac{2^{2I}-1}{SNR}$ and therefore there will be no transmission from the relay during the second time slot. If we denote the channel gain between source$\rightarrow$relay$\rightarrow$destination as g then the mutual information between source and destination can be written as

$$I_d = \frac{1}{2} \log (1 + SNRg). \tag{8}$$

Here 2 in the denominator reflects the fact that two time slots are taken to convey information from source to the destination. The pdf of g for a certain required mutual information I can be written as [24]

$$\begin{aligned} p_g(g) = {} & p_{g|relayfails}(g) Pr(relayfails) \\ & + p_{g|relaypass}(g) Pr(relaypass), \end{aligned} \tag{9}$$

where $Pr(relayfails)$ can be found as

$$Pr(I_r < I) = Pr\left(\frac{1}{2}\log\left(1 + \left(1 - \alpha_n^*\right) P |h_{s_n r}|^2\right) < I\right), \tag{10}$$

after some mathematical manipulations $P(relayfails)$ can be written as

$$Pr(relayfails) = 1 - e^{-\lambda \frac{2^{2I}-1}{SNR}}, \tag{11}$$

and

$$p_{g|relayfails}(g) = \delta(g), \tag{12}$$

because when relay fails to decode the information then it does not transmit anything to the destination in the second time slot. Now we consider the pdf of g when relay has decoded the information correctly in the first time slot. In this case the relay$\rightarrow$destination channel gain will be $\eta\left(|h_{s_n r}|^2 - \frac{2^{2I}-1}{SNR}\right)|h_{RD}|^2$. We need to find pdf of $\left(|h_{s_n r}|^2 - \frac{2^{2I}-1}{SNR}\right)\eta |h_{RD}|^2$ given that $|h_{s_n r}|^2 > \frac{2^{2I}-1}{SNR}$. Putting $\Omega = \left(|h_{s_n r}|^2 - \frac{2^{2I}-1}{SNR}\right)$ and $\Phi = \eta |h_{RD}|^2$ we can see that $p_{\Omega\Phi|\Omega>0}(g) = p_{g||h_{s_n r}|^2 > \frac{2^{2I}-1}{SNR}}(g)$ and hence

$$p_{\Omega\Phi|\Omega>0}(g) = \frac{\lambda^2 \int_0^\infty \Omega^{-1} e^{-\lambda\Omega - \lambda\frac{2^{2I}-1}{SNR}} e^{-\lambda\frac{g}{\eta\Omega}}}{e^{-\lambda\frac{2^{2I}-1}{SNR}}} d\Omega, \tag{13}$$

where we have used the fact that $\Omega \sim \lambda e^{-\lambda\Omega - \lambda\frac{2^{2I}-1}{SNR}}$ and $\Phi \sim \frac{\lambda}{\eta} e^{-\frac{\lambda\Phi}{\eta}}$. Now Eq. 13 can be written as [26]

$$p_{\Omega\Phi|\Omega>0}(g) = \frac{2\lambda^2 K_0\left(2\sqrt{\frac{\lambda^2 g}{\eta}}\right)}{\eta}, \tag{14}$$

where $K_v(x)$ is the modified Bessel function of the second kind and v-th order. Putting Eqs. 14, 12 and 11 into Eq. 9 we get the pdf of g as follows

$$\begin{aligned} p_g(g) = {} & (1 - e^{-\lambda\frac{2^{2I}-1}{SNR}})\delta(g) \\ & + \frac{2\lambda^2 e^{-\lambda\frac{2^{2I}-1}{SNR}} K_0\left(2\sqrt{\frac{\lambda^2 g}{\eta}}\right)}{\eta}, \end{aligned} \tag{15}$$

and the corresponding outage probability is

$$Pr(g < C) = \int_0^C p_g(g)dg \tag{16}$$

$$= (1 - e^{-\lambda C}) - e^{-\lambda C}\left[2\sqrt{\frac{\lambda^2 g}{\eta}}K_1\left(2\sqrt{\frac{\lambda^2 g}{\eta}}\right)\right]\Bigg|_0^C \tag{17}$$

where $C = \frac{2^{2I}-1}{SNR}$, using the fact that $xK_1(x) \to 1$ as $x \to 0$ we can simplify the above expression as follows

$$P_{out} = Pr(g < C) = 1 - 2\sqrt{\frac{\lambda^2 C}{\eta}}e^{-\lambda C}K_1\left(2\sqrt{\frac{\lambda^2 C}{\eta}}\right). \tag{18}$$

3.2 Selection cooperation (SC)

In SC scheme during the second time slot the relay which has the best product of the harvested energy and $h_{r,d}$ will be selected given that it has successfully decoded the source information. As we have found the pdf of the individual source → relay → destination channel gain hence we can apply the methodology of [24] to find the outage probability of the SC scheme. It is important to note that while finding the pdf of channel gain g we considered the harvested energy at the relay as well as the instantaneous channel condition. Therefore, we can say that the channel gain of the best relay path, while considering harvested energy and instantaneous channel conditions, is $\max\limits_{i\in(1\cdots M)} g_i$. The mutual information between source and destination for SC scheme can be written as

$$I_{SC} = \frac{1}{2}\log\left(1 + SNR|h_{s_n d}|^2 + \max_{i\in(1\cdots M)} SNR g_i\right), \tag{19}$$

and the outage probability for a given mutual information I is given as

$$P_{out}^{SC} = Pr(I_{SC} < I) = Pr\left(|h_{s_n d}|^2 + \max_{i\in(1\cdots M)} g_i < C\right) \tag{20}$$

$$= \int_0^C Pr\left(\max_{i\in(1\cdots M)} g_i < C - |h_{s_n d}|^2\right) \times p_{|h_{s_n d}|^2}(|h_{s_n d}|^2)d|h_{s_n d}|^2. \tag{21}$$

Although it is possible to use numerical techniques to solve Eq. 21 however it is difficult to solve the above integration for a general value of M. Therefore, we provide the high SNR expression for outage probability in following Lemma.

Lemma 1 *The high SNR approximation of the outage probability for SC is given as*

$$P_{out}^{SC} \simeq \lambda^{M+1}e^{-\lambda C}\sum_{p=0}^{M}\binom{M}{p}\left(\frac{\lambda}{\eta}\right)^{M-p} \times [A\Theta(M,p,1) - B\Theta(M,p,2) - E\Theta(M,p,3)], \tag{22}$$

where

$$A = e^{(M+1)\left(\ln\left(\frac{\eta}{\lambda^2 x}\right)+\ln(x)\right)}(M+1)^{(p-M)}$$

$$B = \lambda(M-p-1)\,e^{(M+2)\left(\ln\left(\frac{\eta}{\lambda^2 x}\right)+\ln(x)\right)} \times (M+2)^{(p-M)}$$

$$E = \lambda^2(M-p)\,e^{(M+3)\left(\ln\left(\frac{\eta}{\lambda^2 x}\right)+\ln(x)\right)} \times (M+3)^{(p-M)}$$

$$\Theta(M,p,k) = \frac{\Gamma\left(M-p+1, (M+k)\ln\left(\frac{\eta}{\lambda^2 x}\right)\right)}{M+k}\Bigg|_0^C.$$

Proof See the Appendix. □

Hence, a generalized approximation for any M can be found. In Section 4 we will show that the high SNR approximation closely matches with the simulation results.

Remark 1 With the help of above analysis we can also find the outage probability for the best incremental relay scheme. In incremental relaying, cooperation will occur only if the destination is unable to decode information correctly in the first time slot. In this case the outage probability can be written as [25]

$$P_{out} = Pr(|h_{s_n d}|^2 + \max_{i\in(1\cdots M)} g_i < C\,||h_{s_n d}|^2 < C) \times P(|h_{s_n d}|^2 < C) \tag{23}$$

$$P_{out} = Pr(|h_{s_n d}|^2 + \max_{i\in(1\cdots M)} g_i < C). \tag{24}$$

This is exactly same expression as we have in Eq. 20 and therefore we conclude that best incremental relay and selection cooperation scheme performs in same way in terms of outage probability. However, it can be easily inferred that best incremental relay scheme will require lesser transmissions from the relays because transmissions from relays are conditioned on the failure of decoding in the first time slot. However, in selection cooperation the best relay will retransmit in the second time slot irrespective of the decoding result during the first time slot. Albeit, the destination will have to inform about the decoding result through some feedback mechanism.

Remark 2 We can also find the outage probability for the random relay selection scheme with the help of above analysis. It can be shown that the outage probability for the random relay selection scheme is

$$P_{out} = Pr(|h_{s_n d}|^2 + SNRg < C). \tag{25}$$

3.3 All relays cooperation (ARC)

As discussed above, during the first time slot the source broadcasts its signal to the relays and destination and in the rest of time slots the relays transmit. This same time division multiplex (TDM) scheme is also used in [24]. However, our work is different from [24] in that we have considered wireless energy harvesting while no energy harvesting is considered in [24]. The relays are assumed to be harvesting energy from the source only and they do not harvest energy from the other relay transmissions. This is due to following three reasons. First, it can be easily inferred that the strongest signal available for EH is received from the source. This is because the relays can not transmit at higher/equal power than the source power due to the EH efficiency and fading channels among the cooperating nodes. Second, if we consider the EH from the relay transmission then we will have to keep track of the order in which relays transmit in addition to the successful detection at the relays. This will make the outage probability analysis intractable. Thirdly, if we consider the EH from relay transmission then the destination will need the CSI among the relays also in order to perform the maximum ratio combining. This can increase the overhead requirements exponentially for the ARC scheme. Owing to these reasons we ignore the possibility of EH from the relay transmissions. The mutual information between S and D can be written as [24]

$$I_{ARC} = \frac{1}{M+1}\log\left(1+SNR|h_{s_n d}|^2+\sum_{i=1}^{M} SNRg_i\right), \tag{26}$$

and the corresponding outage probability will be

$$P_{out}^{ARC} = Pr\left(|h_{s_n d}|^2 + \sum_{i=1}^{M} g_i < \frac{2^{(M+1)I}-1}{SNR}\right). \tag{27}$$

For a special case when all the time slots and power is used for source to destination we will have $M = 0$ and the outage probability can be written as

$$P_{out,M=0} = Pr\left(|h_{s_n d}|^2 < \frac{(2^I-1)}{SNR}\right), \tag{28}$$

$$P_{out,M=0} = 1 - e^{-\frac{(2^I-1)}{SNR}}. \tag{29}$$

It may appear from Eqs. 26 and 27 that all the relays are forwarding all the time irrespective of the successful detection at the relay. However, this is not the case because the variable g takes into consideration the successful detection at the relay [24]. We already know the cdf of $|h_{s_n d}|^2$ and individual g_i's, however it is difficult to get a general exact answer for all possible values of M. Therefore, first we will find the upper and lower bounds on Eq. 27.

3.3.1 Lower bound

We know that $g_i \geq 0 \; \forall i \in (1\cdots M)$ and $|h_{s_n d}|^2 \geq 0$, therefore $\sum_{i=1}^{M} g_i \leq M \times \max_{i\in(1\cdots M)} g_i$ and hence we can write

$$Pr\left(|h_{s_n d}|^2 + M \times \max_{i\in(1\cdots M)} g_i \leq \frac{2^{(M+1)I}-1}{SNR}\right) \leq Pr\left(|h_{s_n d}|^2 + \sum_{i=1}^{M} g_i \leq \frac{2^{(M+1)I}-1}{SNR}\right). \tag{30}$$

Hence, we have

$$P_{out}^{ARC} \geq \int_0^{C_1} Pr\left(\max_{i\in(1\cdots M)} g_i < \frac{C_1-|h_{s_n d}|^2}{M}\right) \times p_{|h_{s_n d}|^2}(|h_{s_n d}|^2) d|h_{s_n d}|^2, \tag{31}$$

where $C_1 = \frac{2^{(M+1)I}-1}{SNR}$. Further, we can follow the steps carried out in B subsection to get the high SNR approximation for lower bound.

3.3.2 Upper bound

We know that $\max_{i\in(1\cdots M)} g_i \leq \sum_{i=1}^{M} g_i$, and therefore we can write

$$P_{out}^{ARC} \leq Pr\left(|h_{s_n d}|^2 + \max_{i\in(1\cdots M)} g_i \leq C_1\right), \tag{32}$$

where equality will be true when only one g_i has nonzero value and all of others have zero value.

Remark 3 We cannot solve Eq. 27 for general value of M because it involves (i) computation of moment generating function (MGF) of g_i's, (ii) taking inverse Laplace transform of the multiplication of MGFs of all g_i and $|h_{s_n d}|^2$, (iii) integration of the inverse Laplace transform from 0 to C_1. To the best of our knowledge (27) cannot be written in the form of known mathematical functions. Although Eq. 27 cannot be solved analytically, it is possible to judge the looseness of the bounds from the actual performance because there are two sources for the deviation of the bounds from the actual performance. The first source is due to the representation of $\sum_{i=1}^{M} g_i$ by $M \times \max_{i\in(1\cdots M)} g_i$ (in the case of lower bound) and $\sum_{i=1}^{M} g_i$ by $\max_{i\in(1\cdots M)} g_i$ (in the case of upper bound). The second source is due to representation of $e^{kx} = 1 + kx$. As M increases the deviation increases because the representations of $\sum_{i=1}^{M} g_i$ by $M \times \max_{i\in(1\cdots M)} g_i$ (in the case of lower bound) and by $\max_{i\in(1\cdots M)} g_i$ (in the case of upper bound) become lose. This looseness translate into deviation of the bounds from the actual performance. In a similar manner as λ increases the representations of $e^{-\lambda x}$ by $1-\lambda x$, $e^{\lambda x}$ by $1+\lambda x$ becomes weak. Therefore, the deviation of the bounds for higher λ is higher than at smaller λ. We introduce two metrics (i) upper bound difference (UBD) and (ii) lower bound difference (LBD) to judge the tightness of the bounds. UBD and LBD represent the absolute difference of the upper and lower bounds from the actual value of the SNR to achieve a certain outage probability. UBD (LBD) for a given outage probability 10^{-x} is mathematically defined as $UBD = |SNR_{act} - SNR_{ub}|$ ($LBD = |SNR_{act} - SNR_{lb}|$) where SNR_{act} is the actual SNR required to achieve 10^{-x} outage probability while $SNR_{ub}(SNR_{lb})$ is the SNR provided by upper (lower) bound to achieve 10^{-x} outage probability. Further, a quantitative comparison of the tightness of the bounds in the form of UBD and LDB is provided in simulation results section in Figs. 4 and 5.

As it has been observed that the bounds become lose with increasing number of relays. Therefore, we provide an approximation in the following for larger number of EH relays.

3.3.3 Approximation for larger M

In this approximation, we use the central limit theorem. Therefore, we will use $\sum_{i=1}^{M} g_i = Z_M$, where Z_M is a Gaussian variable with mean $\omega = M\overline{g_i}$and variance $\sigma^2 = M\sigma_{g_i}^2$ $\forall i \in (1\cdots M)$. Since all the g_i's follow same distribution, therefore $\overline{g_1} = \overline{g_2} = \cdots \overline{g_M} = \overline{g}$. The values of ω, σ and outage probability are provided in the following Lemma.

Lemma 2 *The outage probability of ARC scheme for large M is given as follows*

$$P_{out}^{ARC} \simeq \frac{1}{2}\left[1 - e^{-\lambda C_1}\right] + \frac{1}{2}e^{\omega - C_1}\left[e^{\lambda x} erf\left(\frac{x}{\sigma\sqrt{2}}\right)\right.$$
$$\left. - e^{\frac{\lambda^2\sigma^2}{2}} erf\left(\frac{x}{\sigma\sqrt{2}} - \frac{\lambda\sigma}{\sqrt{2}}\right)\right]\Bigg|_{-\omega}^{C_1-\omega}, \quad (33)$$

where $\omega = M\overline{g_i}$, $\sigma^2 = M\sigma_{g_i}^2$ and

$$\overline{g} = e^{\frac{\lambda}{2\eta}}\lambda^{-1}\left[2W_{-2,\frac{1}{2}}\left(\frac{\lambda}{\eta}\right) - \frac{1}{2}W_{-1,\frac{1}{2}}\left(\frac{\lambda}{\eta}\right)\right.$$
$$\left. + \frac{1}{2}\sqrt{\frac{\lambda}{\eta}}W_{-\frac{3}{2},0}\left(\frac{\lambda}{\eta}\right) + \sqrt{\frac{\lambda}{\eta}}W_{-\frac{3}{2},1}\left(\frac{\lambda}{\eta}\right)\right], \quad (34)$$

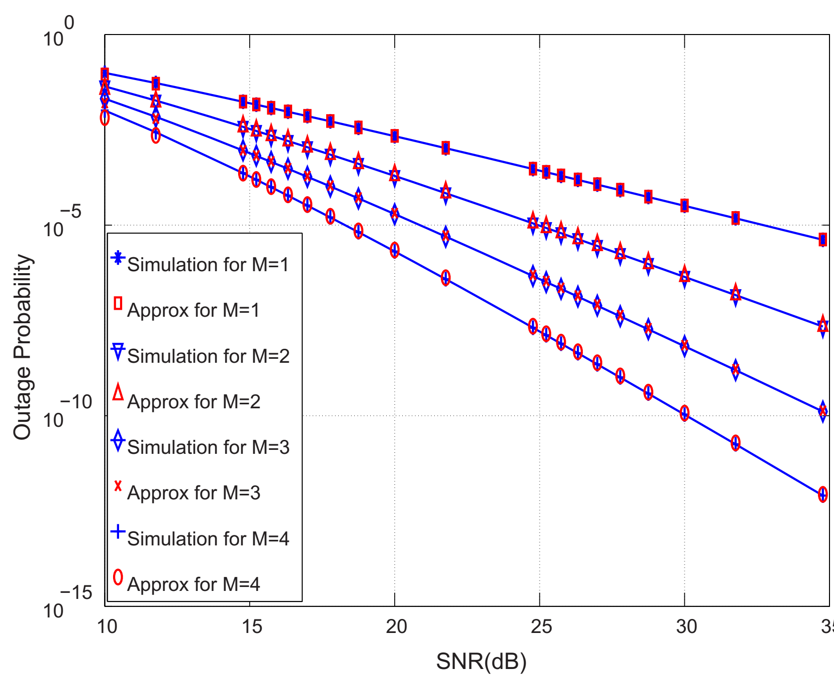

Fig. 2 Outage probability for the SC scheme

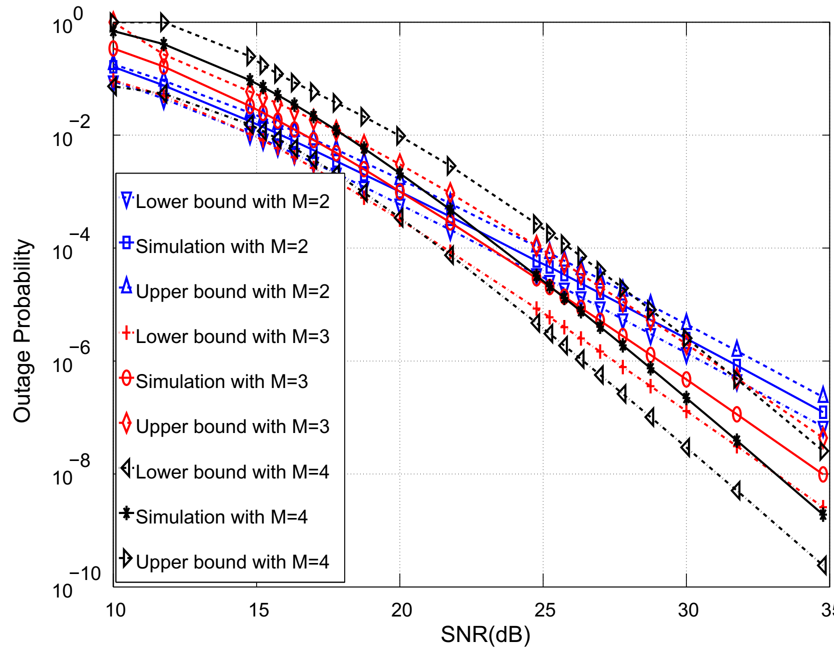

Fig. 3 Outage probability for the ARC scheme

$$\sigma_g^2 = 12e^{\frac{\lambda}{2\eta}}\lambda^{-2}W_{-3,\frac{1}{2}}\left(\frac{\lambda}{\eta}\right) - e^{\frac{\lambda}{2\eta}}\lambda^{-2}W_{-2,\frac{1}{2}}\left(\frac{\lambda}{\eta}\right)$$
$$+2e^{\frac{\lambda}{2\eta}}\sqrt{\frac{1}{\lambda^3\eta}}W_{-\frac{5}{2},0}\left(\frac{\lambda}{\eta}\right)$$
$$+6e^{\frac{\lambda}{2\eta}}\sqrt{\frac{1}{\lambda^3\eta}}W_{-\frac{5}{2},1}\left(\frac{\lambda}{\eta}\right) - \overline{g}^2. \quad (35)$$

Proof See the Appendix. □

4 Simulation results

Simulations are performed in MATLAB. For the simulations, we assume $\eta = 1, I = 1$ and number of relays varies from 1 to 4. The SNR is varied from $10 - 35$ dB. First, we will discuss the outage performance of the SC and ARC scheme then we will present the variations of actual and approximated performance of the ARC scheme. In Fig. 2, we show the exact and approximate outage probability for the SC schemes with $\lambda = 1$. It can be observed

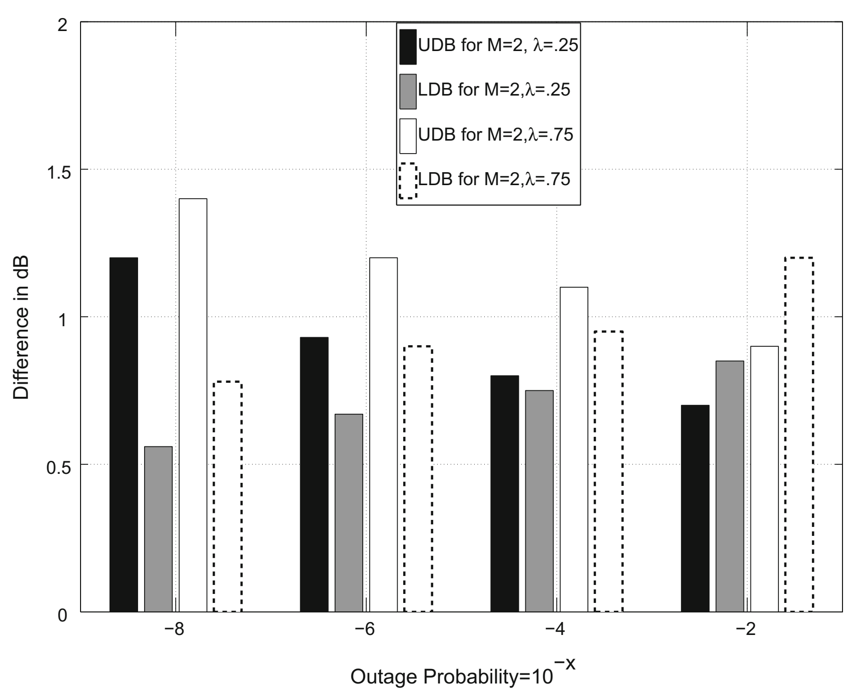

Fig. 4 UDB and LDB for M = 2

that the approximated results closely match the simulation results. This is because at higher SNR the (36)–(38) approximations are tight. As the approximations are quite tight therefore we do not provide the error in the approximated performance for the SC scheme because it is negligibly small.

The outage performance for ARC scheme is shown in Fig. 3. It can be verified that lower and upper bounds provide the floor and ceiling of the ARC outage performance. Further, we see that SC outperforms ARC. This is because in SC scheme for half of the transmission time the destination receives signal from the source and for the other half it receives signal from the best chosen relay while for ARC scheme more than half of the time is allocated to the relay transmission which are comparatively weak with respect to source transmission. Hence, a degradation in outage probability is experienced. Another important observation is that for small *SNR* the ARC system with smaller M performs better than the ARC system with higher M. This is because with increasing number of relays the $\frac{(2^{(M+1)I}-1)}{SNR}$ factor in Eq. 27 increases exponentially while the left side increase linearly. However, with increase in *SNR* the exponential effect in $\frac{(2^{(M+1)I}-1)}{SNR}$ is balanced by division by a higher value of *SNR* and therefore we see that at higher power levels the performance of the ARC scheme is better for higher M as compared to the smaller M.

Now, we show the effect of the λ, M on the lower and upper bounds of the outage probability for ARC scheme in Figs. 4 and 5. For this purpose we use $\lambda = (.25, .75)$ and $M = (2, 4)$. The result we present shows the absolute difference of the upper and lower bounds from the actual value of the SNR required to achieve a certain outage probability. It means that if for 10^{-x} outage probability the actual required SNR is SNR_{act} while we get SNR_{lb} from the lower bound expression then $LBD = |SNR_{act} - SNR_{lb}|$ for 10^{-x} outage probability. Similarly, UBD represent the difference of the actual required SNR from the upper bound estimate that is $UBD = |SNR_{act} - SNR_{ub}|$. It can be observed from Figs. 4 and 5 that as outage probability decreases the UBD increases while LBD decreases. In addition, the values of UBD and LBD are higher for $M = 4$ as compared to $M = 2$. These two behaviors can be explained as follows. As smaller outage probability is achieved at higher SNR hence almost all relays provide good signal quality at the destination and therefore the lower bound provides a better estimate of the performance at the smaller outage probabilities. This is because in lower bound estimate we have assumed that all the channel from the source→ relay → destination are the best channels. The situation is reversed at the higher outage probabilities. In this case the SNR is smaller due to which we can say that the destination may receive good quality signal from a small subset of the total number of relays and hence the upper bound provides a better estimate in this situation. The higher value of UBD and LBD for higher number of relays can be explained by the fact that the approximation $\sum_{i=1}^{M} g_i \leq M \times \max_{i\in(1\cdots M)} g_i$ used to find the lower bound and the approximation $\sum_{i=1}^{M} g_i \leq \max_{i\in(1\cdots M)} g_i$ becomes weak as the number of relays increase. The dependence of the LBD

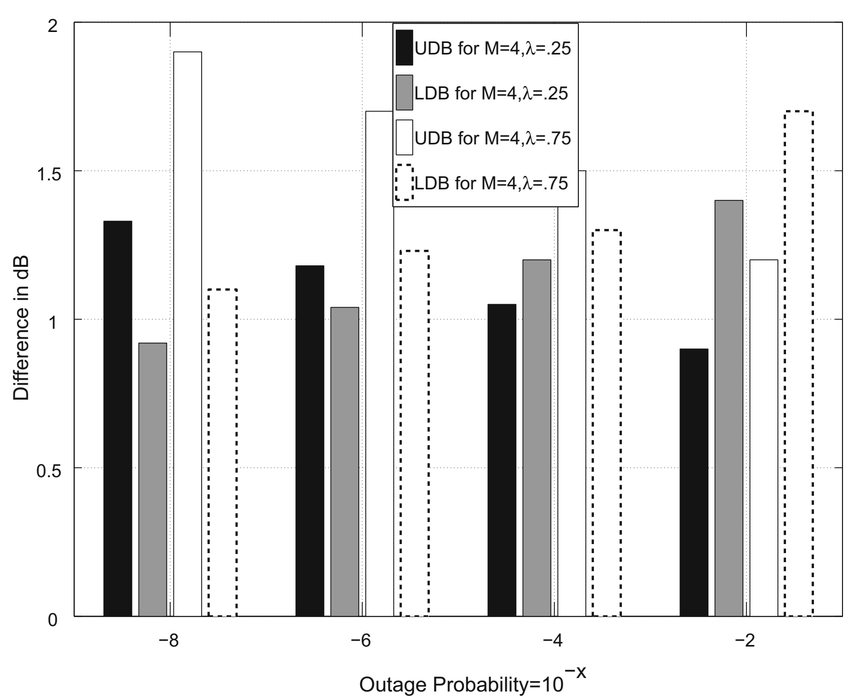

Fig. 5 UDB and LDB for M = 4

and UBD on λ can also be observed from Figs. 4 and 5. It can be observed that as λ decreases the lower and upper bounds estimate become more accurate because the estimation $e^{-x} = 1 - x$ becomes more accurate with decreasing x.

5 Conclusions

In this paper, we have approximated the outage probability of cooperative communication when EH relays are used for cooperation. We have considered the case of CSI available at the source, in the form of SC scheme, and the case in which it is unavailable the source, in the form of ARC scheme. Simulation results show that analytical results closely approximates the performance of the SC and ARC scheme. Further, it is seen that SC scheme outperforms ARC scheme in terms of outage probability. The main reason for this superiority of SC scheme is the higher amount of time over which transmission is received from the source.

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Compliance with Ethical Standards

Conflict of interests The authors declare that they have no conflict of interest.

Appendix

Proof of Lemma 1 We know the cdf of individual g_i's and since they are independent therefore we can write the outage probability for SC scheme as

$$\int_0^C \left[1 - 2\sqrt{\frac{\lambda^2(C-|h_{s_n d}|^2)}{\eta}} e^{-\lambda(C-|h_{s_n d}|^2)} \times K_1\left(2\sqrt{\frac{\lambda^2(C-|h_{s_n d}|^2)}{\eta}}\right)\right]^M \times \lambda e^{-\lambda|h_{s_n d}|^2} d|h_{s_n d}|^2. \quad (36)$$

After some mathematical manipulations, the above integral can be written as

$$P_{out} = e^{-\lambda C}\int_0^C \left[1 - e^{-\lambda x} 2\sqrt{\frac{\lambda^2 x}{\eta}} K_1\left(2\sqrt{\frac{\lambda^2 x}{\eta}}\right)\right]^M \times \lambda e^{\lambda x} dx. \quad (37)$$

For high SNR, we use following approximations

$$xK_1(x) \simeq 1 + \frac{x^2}{2}\ln\frac{x}{2} \quad (38)$$

$$e^{kx} \simeq 1 + kx \quad (39)$$

$$(1+x)^k \simeq 1 + kx \quad (40)$$

Hence Eq. 37 can be simplified to

$$P_{out}^{SC} \simeq \lambda^{M+1} e^{-\lambda C}\int_0^C x^M \left[1 + \frac{\lambda}{\eta}(1-\lambda x)\ln\left(\frac{\eta}{x\lambda^2}\right)\right]^M \times (1+\lambda x)dx, \quad (41)$$

we can use binomial expansion to get

$$P_{out}^{SC} \simeq \lambda^{M+1} e^{-\lambda C}\sum_{p=0}^{M}\binom{M}{p}\int_0^C \left(\frac{\lambda}{\eta}\right)^{M-p} \times (1-(M-p)(\lambda x))\, x^M \ln^{M-p} \times \left(\frac{\eta}{x\lambda^2}\right)(1+\lambda x)\, dx, \quad (42)$$

the result of the inner integration can be written as

$$(\frac{\lambda}{\eta})^{M-p}[A\Theta(M,p,1) - B\Theta(M,p,2) - D\Theta(M,p,3)]. \quad (43)$$

□

By putting Eq. 43 into Eq. 42 Lemma 2 is proved.

Proof of Lemma 2 The pdf of g can be found using Eq. 18

$$p_g(x) = \frac{\partial Pr(g<x)}{\partial x} = e^{-\lambda x}\left[\lambda K_1\left(\sqrt{\frac{4\lambda^2 x}{\eta}}\right)\sqrt{\frac{4\lambda^2 x}{\eta}} - \sqrt{\frac{\lambda^2}{\eta x}} K_1\left(\sqrt{\frac{4\lambda^2 x}{\eta}}\right) + K_0\left(\sqrt{\frac{4\lambda^2 x}{\eta}}\right)\frac{\lambda^2}{\eta} + K_2\left(\sqrt{\frac{4\lambda^2 x}{\eta}}\right)\frac{\lambda^2}{\eta}\right], \quad (44)$$

and $\overline{g}$ and σ_g^2 are given by

$$\overline{g} = \int_0^\infty x p_g(x) dx, \quad (45)$$

$$\sigma_g^2 = \int_0^\infty x^2 p_g(x) dx - \overline{g}^2, \quad (46)$$

Now we can use following identity from [26]

$$\int_0^\infty x^{\mu-\frac{1}{2}} e^{-\gamma x} K_{2\nu}(2\beta\sqrt{x})dx = \frac{\Gamma(\mu+\nu+\frac{1}{2})\Gamma(\mu-\nu+\frac{1}{2})}{2\beta} e^{\frac{\beta^2}{2\gamma}} \gamma^{-\mu} W_{-\mu,\nu}\left(\frac{\beta^2}{\gamma}\right), \quad (47)$$

where $W_{\alpha,\beta}(x)$ is the Whittaker function. Putting Eq. 44 into Eqs. 45, 46 and using Eq. 47 we can get Eqs. 34

and 35. Now we can write the right hand side of Eq. 27 as follows

$$P_{out}^{ARC} \simeq \int_0^{C_1} \frac{1}{2}\left[1 + erf\left(\frac{C_1 - |h_{s_n d}|^2 - \omega}{\sigma\sqrt{2}}\right)\right] \times \lambda e^{-\lambda |h_{s_n d}|^2} d|h_{s_n d}|^2. \quad (48)$$

After simplification [27] we can get Eq. 33. □

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